Global sensitivity analysis of bistable energy harvesting systems via Sobol indices

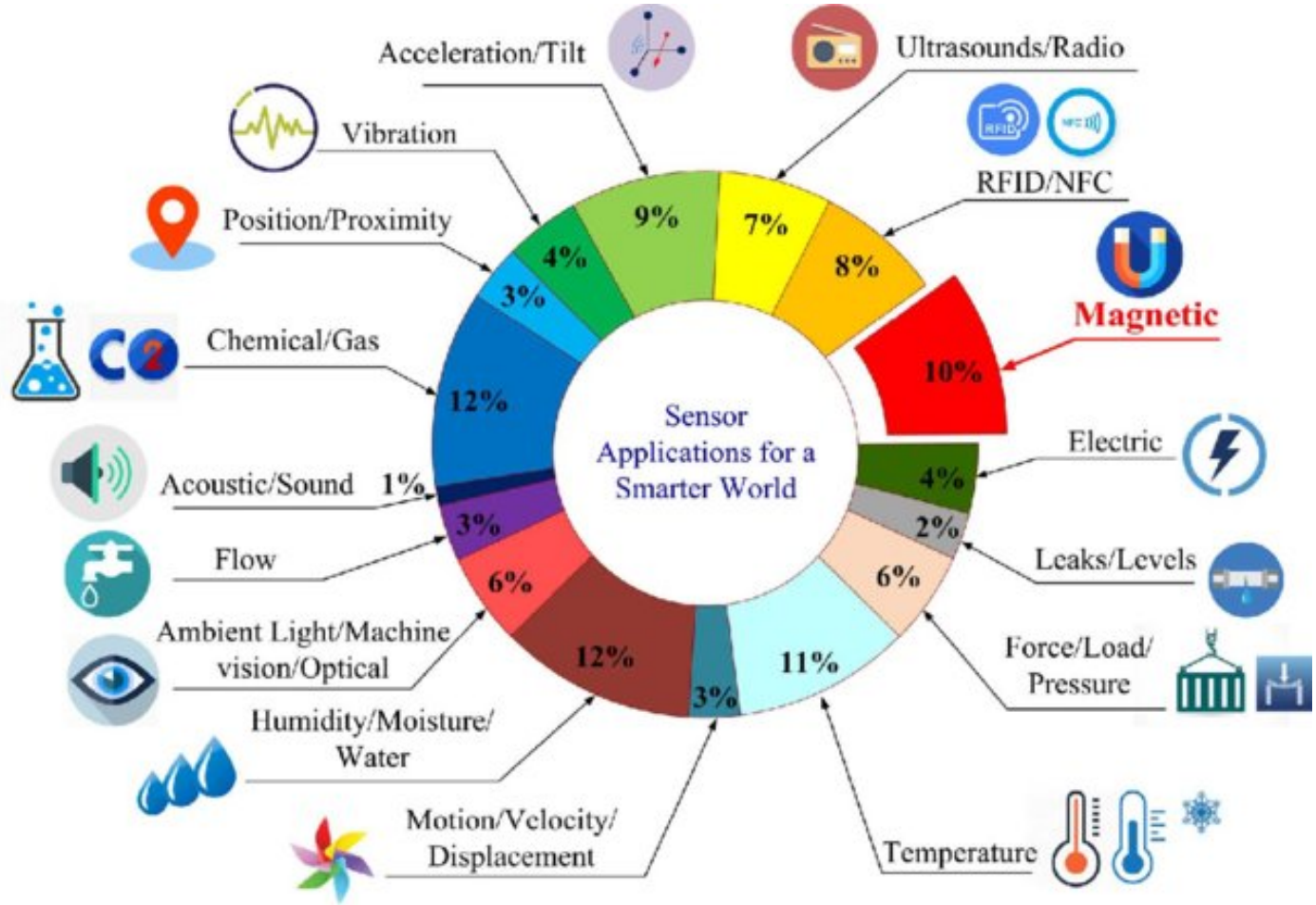
João Pedro C. V. Norenberg[1] **Americo Cunha Jr**[2]
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SR40 - Stochastic Resonance 40 years

Perugia, Italy, September 13-15, 2021

IoT will demand more than a trillion sensors by 2025



L. Xuyang et al., *Overview of Spintronic Sensors With Internet of Things for Smart Living.* **IEEE Transactions on Magnetics**, 0018-9464, 2019.

*Picture from this reference.

Energy and environmental challenges are huge!

Up to 78 million batteries will be discarded daily by 2025, researchers warn

An EU-funded project makes it plain: batteries need to live longer than the Internet of things (IoT) devices they power. A position paper published by the project outlines the key actions to power IoT devices reliably and sustainably.

CLIMATE CHANGE AND ENVIRONMENT

ENERGY

SCIENTIFIC ADVANCES

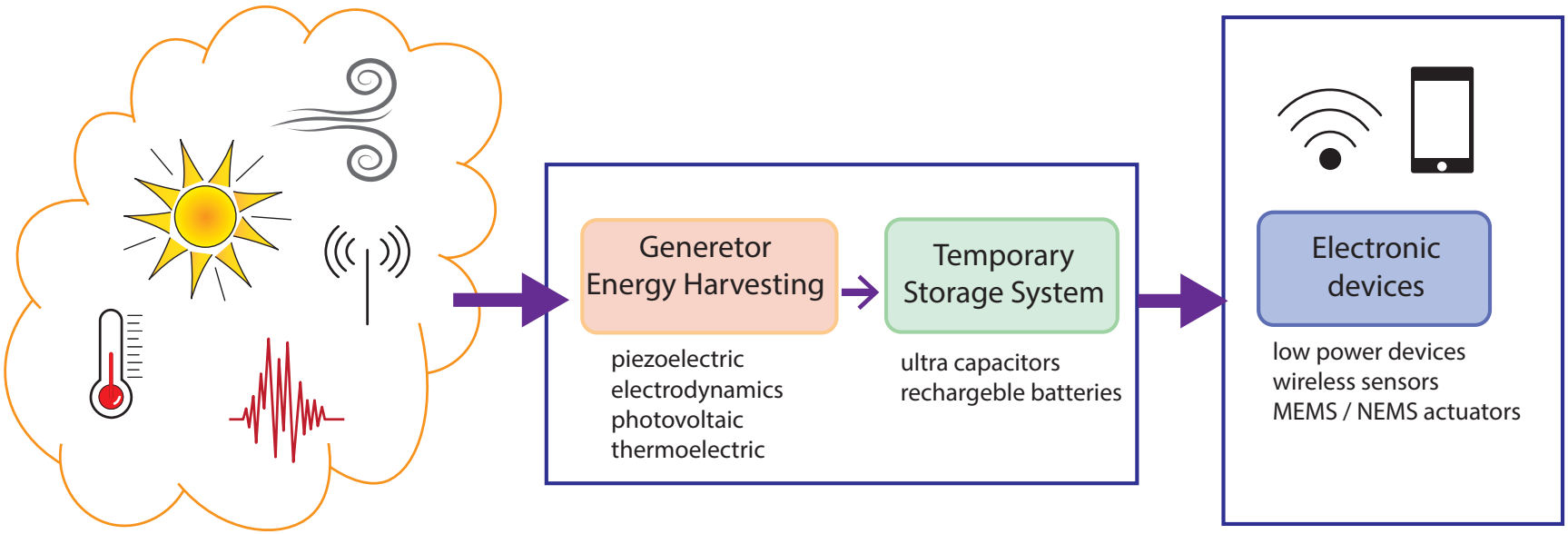


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CORDIS — European Commission, *Up to 78 million batteries will be discarded daily by 2025, researchers warn (July 23, 2021)*

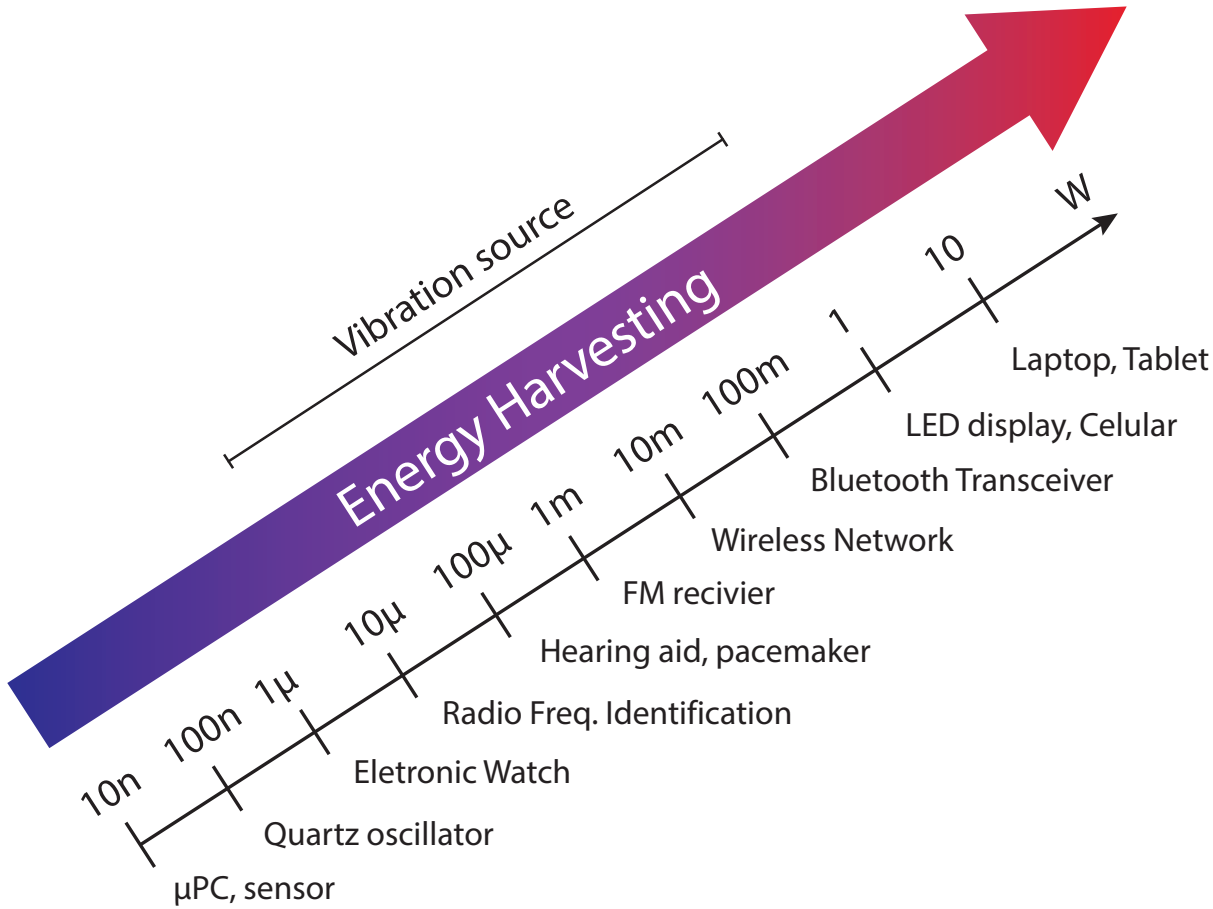
Energy Harvesting concept

ENERGY HARVESTING AS ALTERNATIVE FOR MICROPOWERING



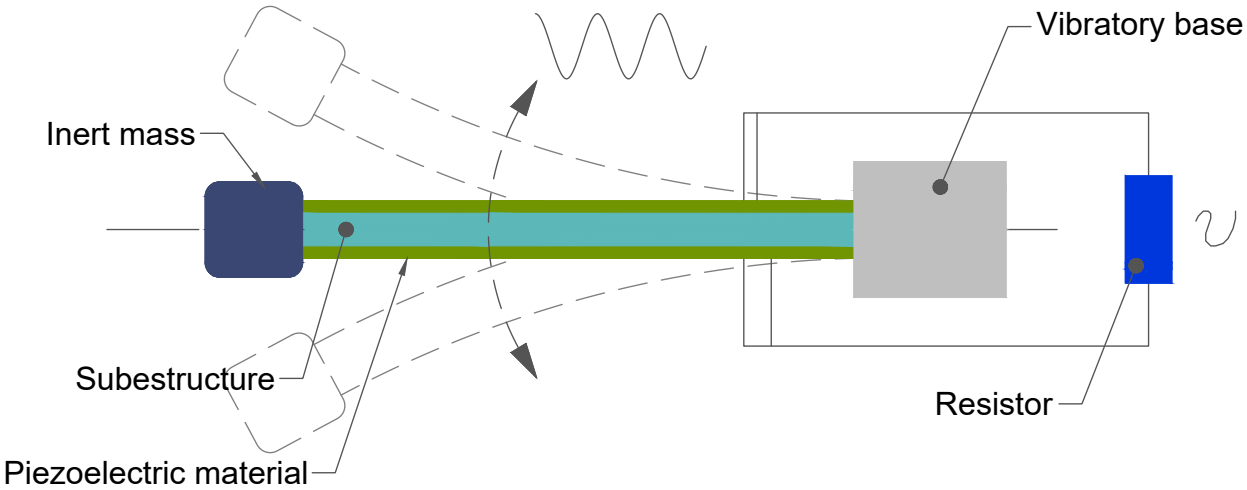
- Capture wasted energy from external sources
- Store this wasted energy for future use
- Use the stored energy to supply other devices

Vibration Energy Harvesting Scale



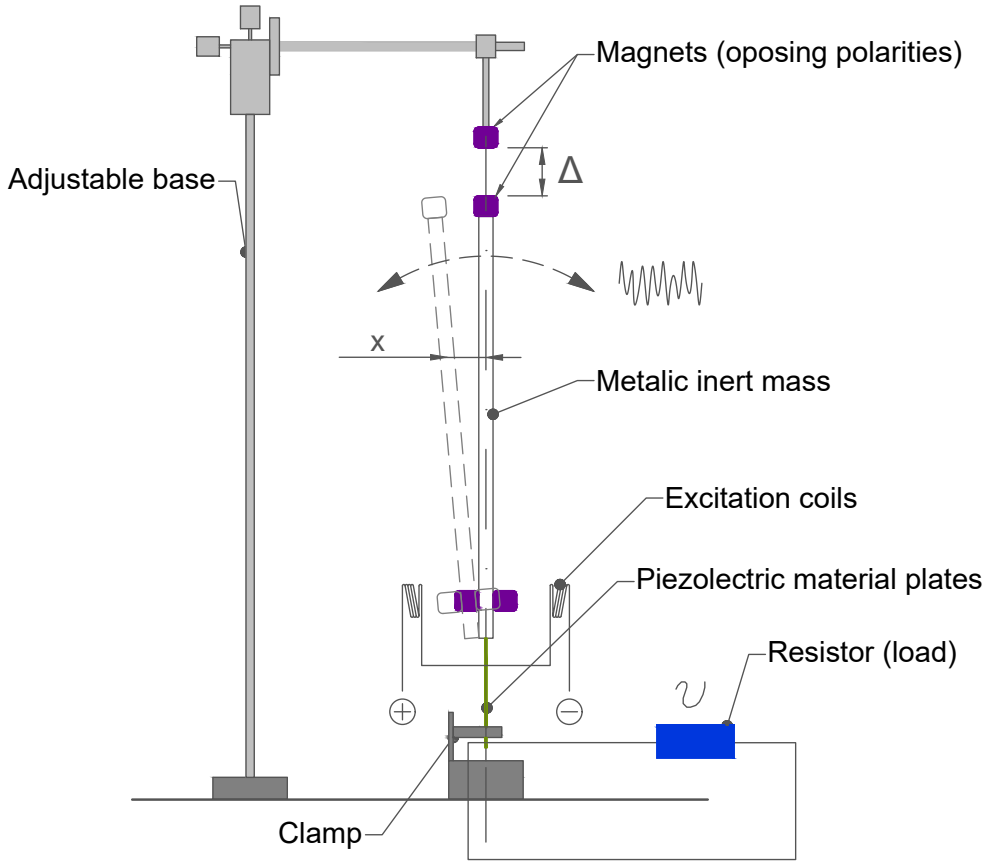
Adapted from: https://passive-components.eu/energy-harvesting-is-not-fiction-anymore-2/

Monostable system driven by regular signal



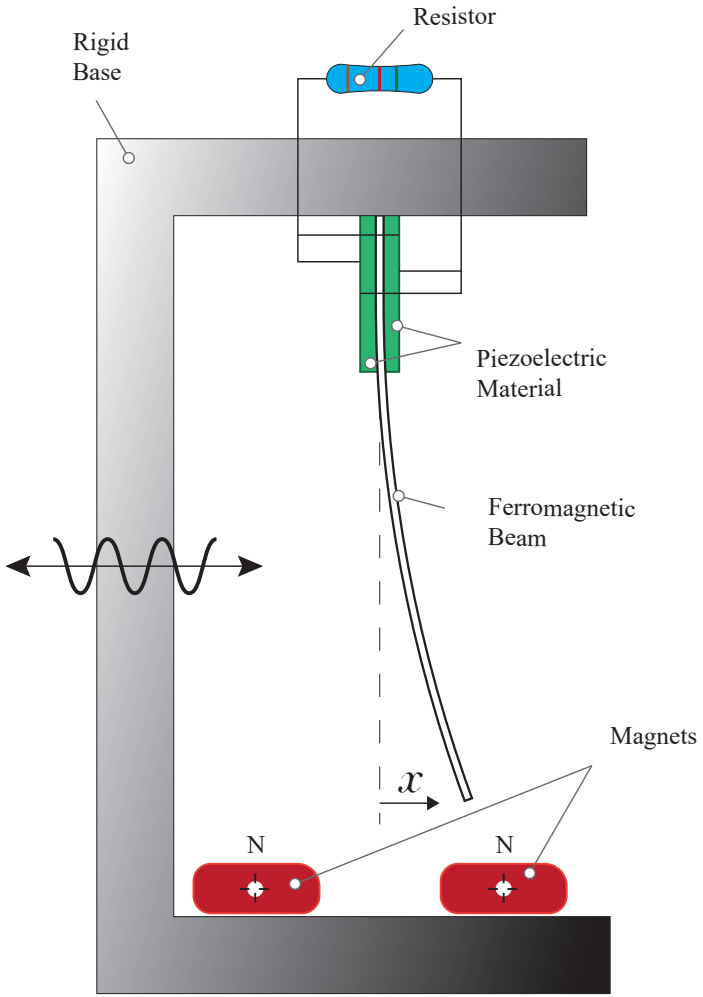
S. Roundy, P. K. Wright and J. Rabaey, A study of low level bibrations as a power source for wireless sensor nodes. **Computer Communications**, 26: 1131-1144, 2003.

Bistable system driven by a noisy signal



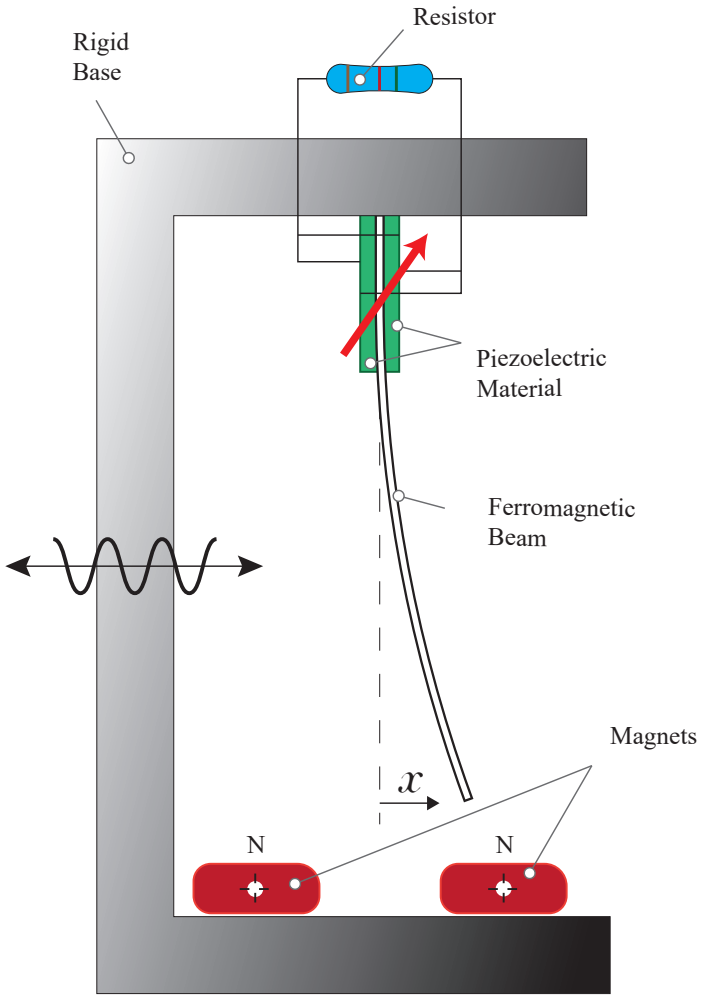
F. Cottone, H. Vocca and L. Gammaitoni, Nonlinear energy harvesting. **Physical Review Letters**, 102: 080601, 2009.

Bistable system driven by regular signal (linear transduction)



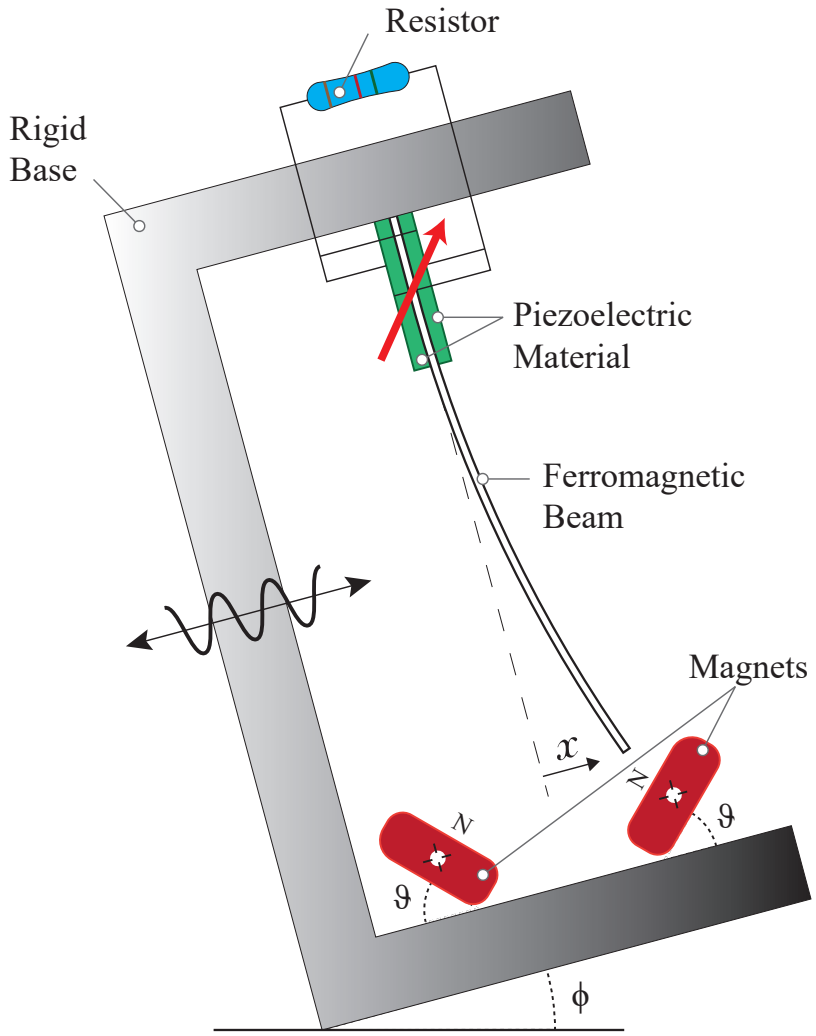
A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting.* **Applied Physics Letters**, 2009.

Bistable system driven by regular signal (nonlinear transduction)



A Triplett, DD Quinn, *The effect of non-linear piezoelectric coupling on vibration-based energy harvesting*, **Journal of Intelligent Material Systems and Structures**, 2009.

Asymmetric bistable system driven by regular signal (nonlinear transduction)

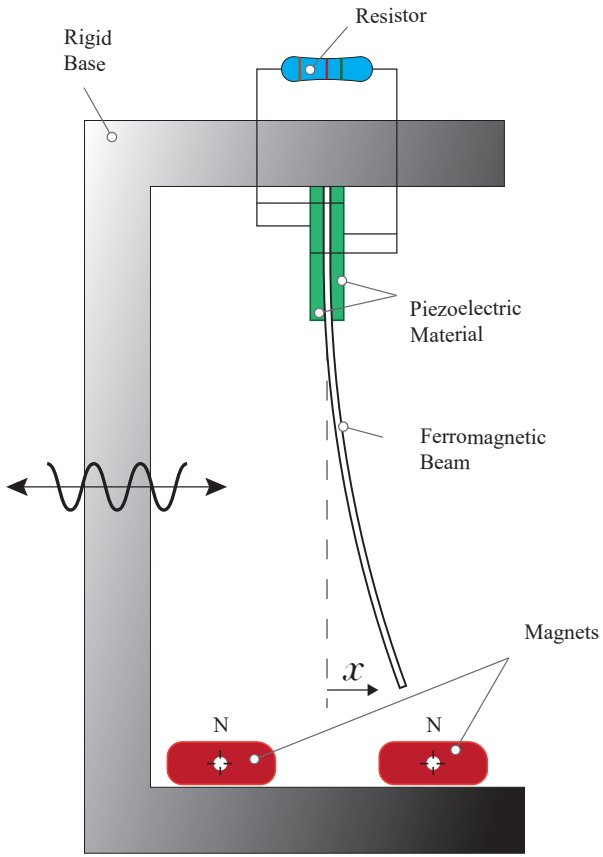


W. Wang, . Cao, C. R. Bowen, D. J. Inman and J. Lin, *Performance enhancement of nonlinear asymmetric bistable energy harvesting from harmonic, random and human motion excitations*, **Applied Physics Letters**, 2018.

Research objectives

- Study variability effects on a asymmetric bistable energy harvester with nonlinear transduction, seeking to find paths to optimize the energy recovering process
- Apply global sensitivity analysis to:
  1. extract features of the dynamic behavior;
  2. identify the most sensitivity parameters.

Bistable harvester: symmetric + linear coupling

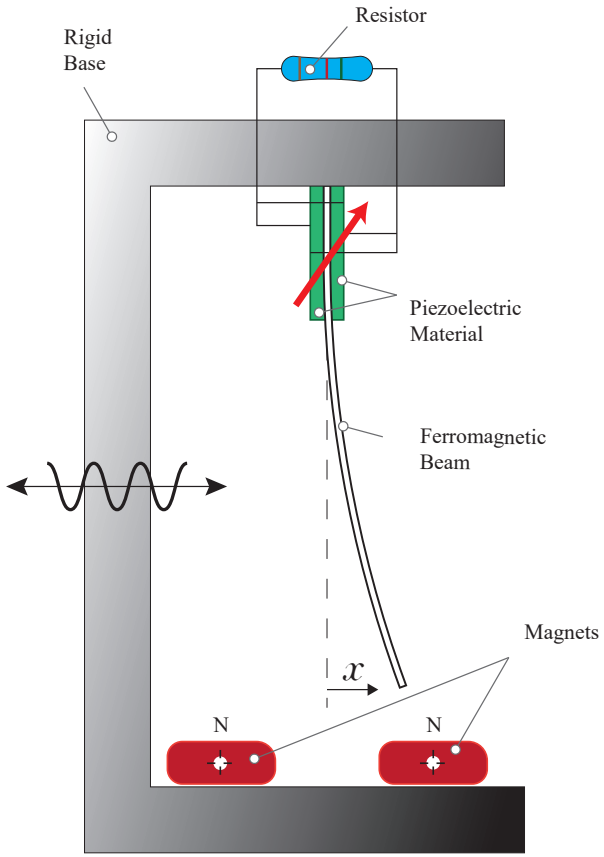


$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \chi v = f\cos(\Omega t)$$

$$\dot{v} + \lambda v + \kappa\dot{x} = 0$$

$+$ initial conditions

Bistable harvester: symmetric + nonlinear coupling



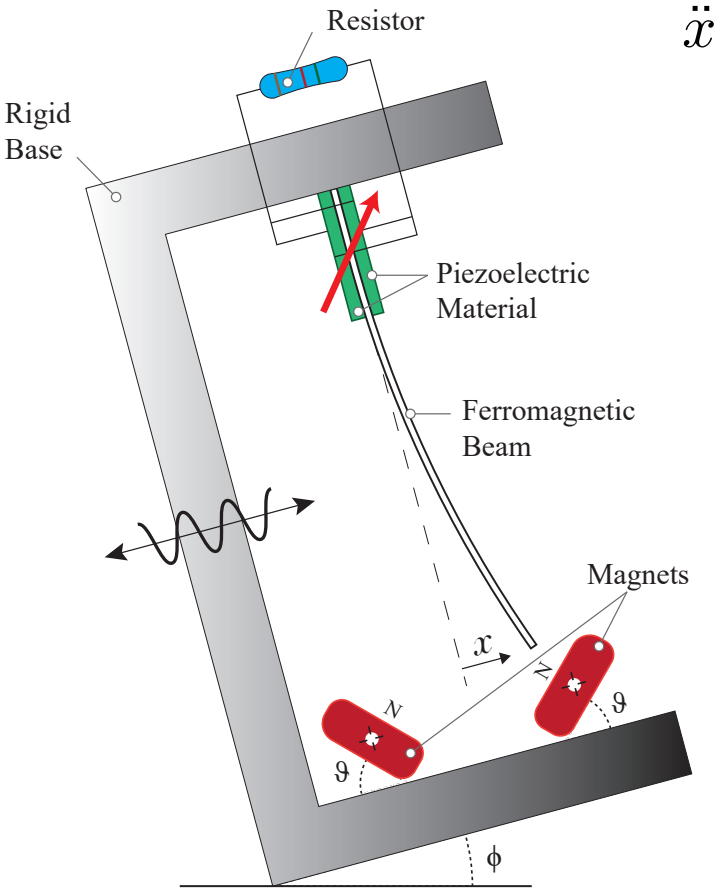
$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1-x^2) - \widehat{\Theta}(x)\chi v = f\cos(\Omega t)$$

$$\dot{v} + \lambda v + \widehat{\Theta}(x)\kappa\dot{x} = 0$$

$+$ initial conditions

$$\widehat{\Theta}(x) = (1 + \beta|x|)$$

Bistable harvester: asymmetric + nonlinear coupling



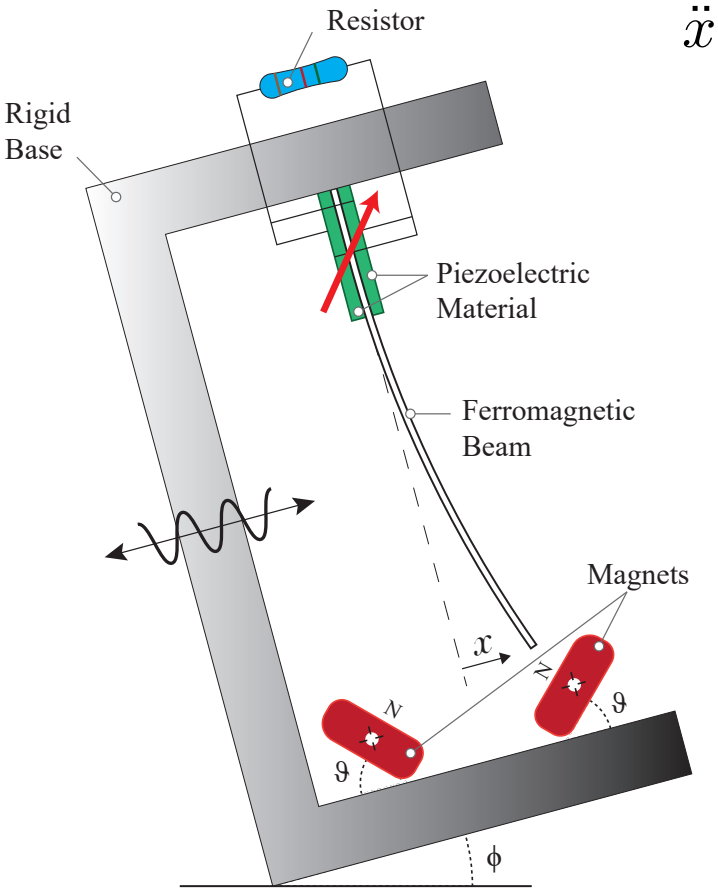
$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 + 2\delta x - x^2) - \widehat{\Theta}(x)\chi v = f\cos(\Omega t) + p\sin(\phi)$$

$$\dot{v} + \lambda v + \widehat{\Theta}(x)\kappa\dot{x} = 0$$

$+$ initial conditions

$$\widehat{\Theta}(x) = (1 + \beta|x|)$$

Bistable harvester: asymmetric + nonlinear coupling



$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 + 2\delta x - x^2) - \widehat{\Theta}(x)\chi v = f\cos(\Omega t) + p\sin(\phi)$$

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$+$ initial conditions

$$\widehat{\Theta}(x) = (1 + \beta|x|)$$

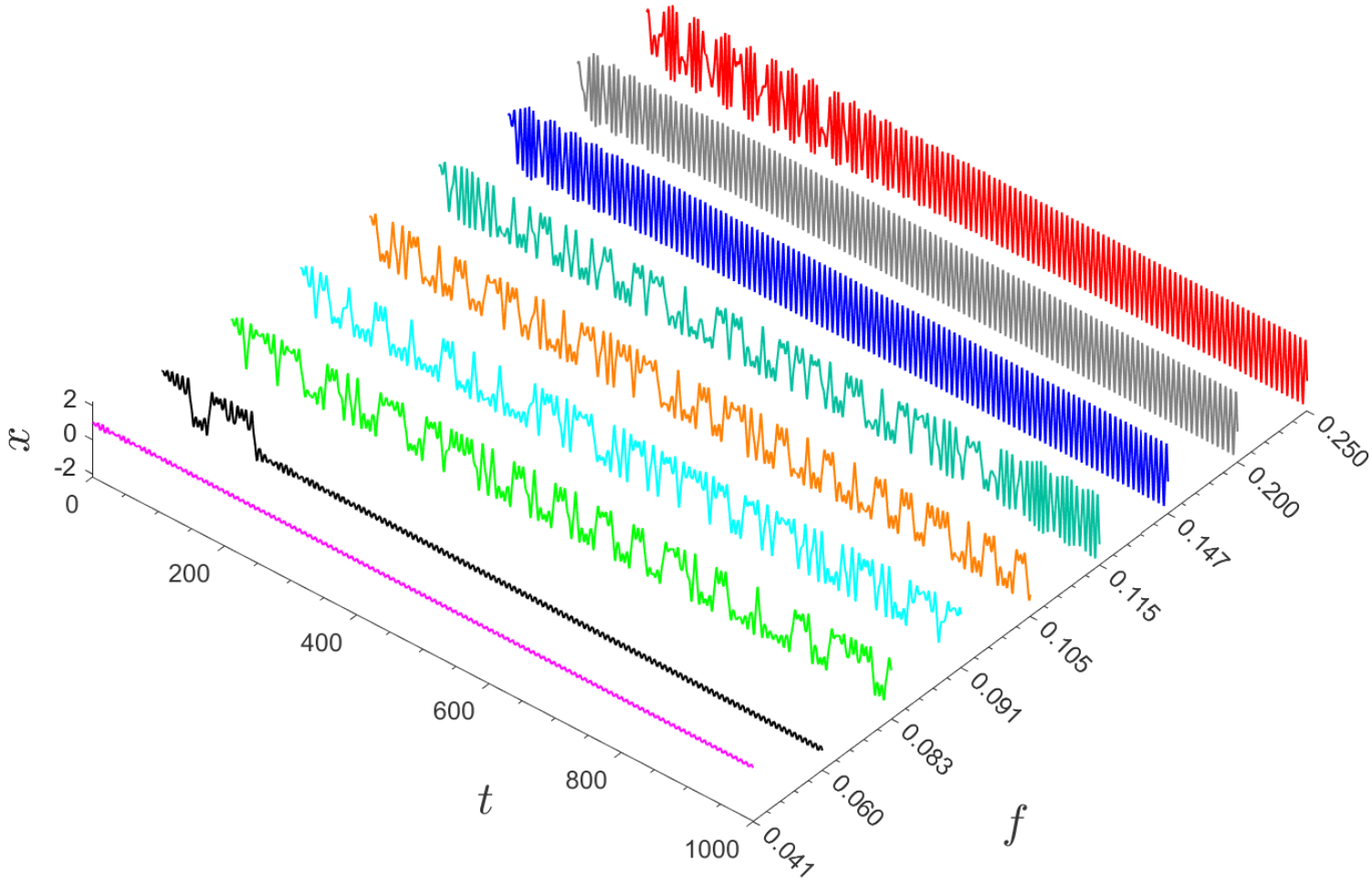
$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} \lambda\, v(t)^2 \; dt$$

Dynamic animation (symmetric + linear coupling)

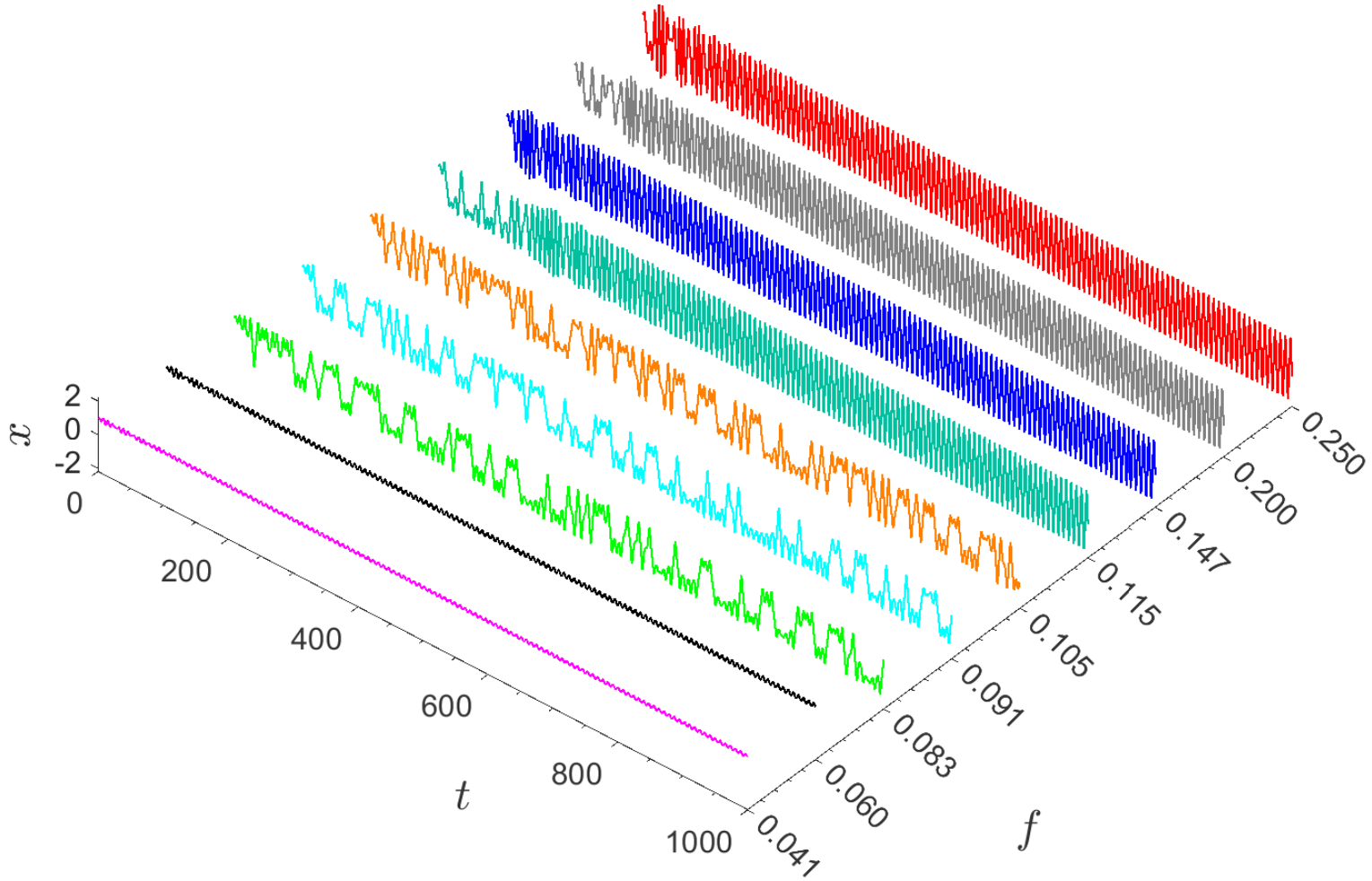
Dynamic animation (symmetric + nonlinear coupling)

Dynamic animation (asymmetric + nonlinear coupling)

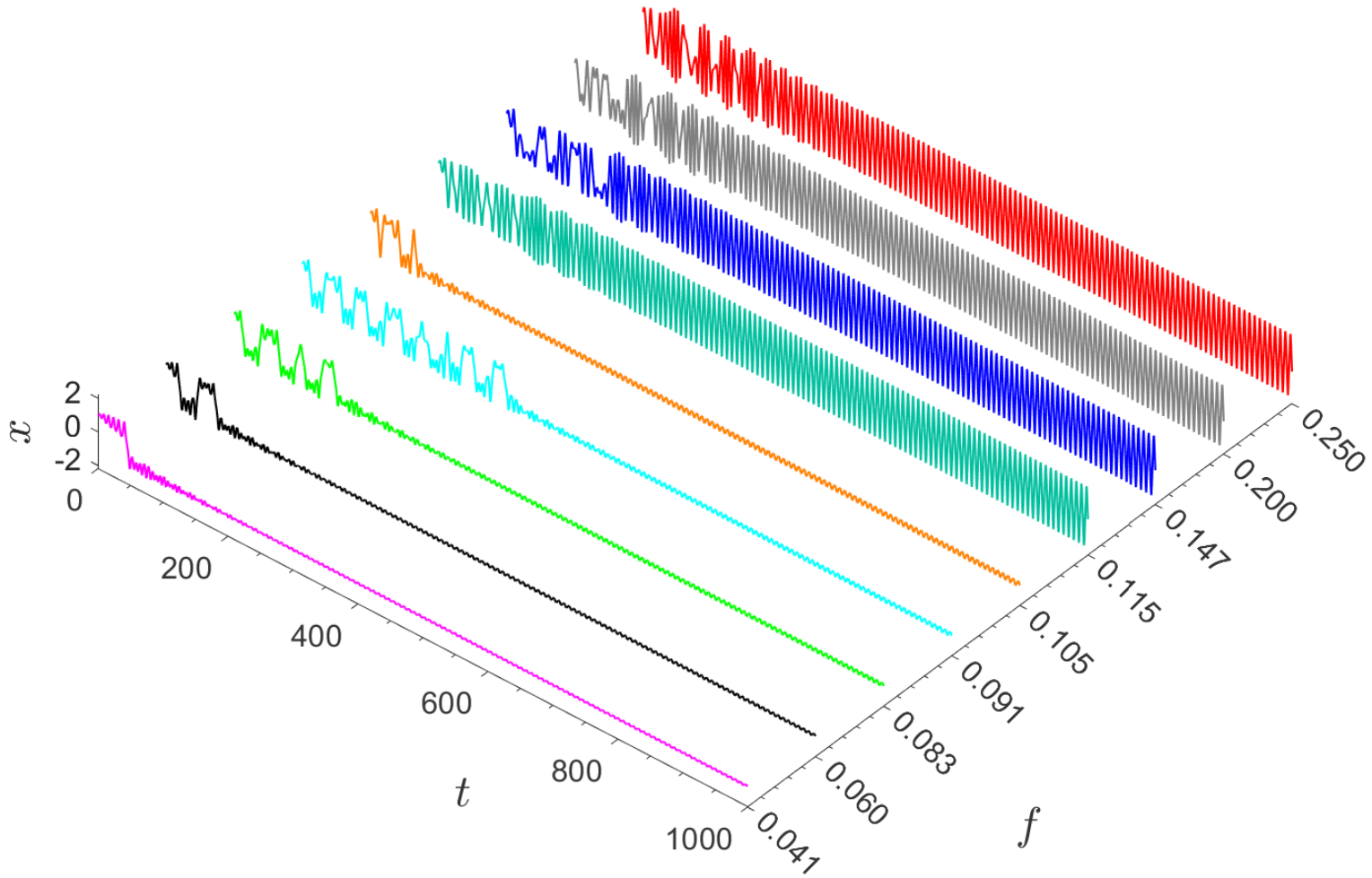
Time series (symmetric + linear coupling)



Time series (symmetric + nonlinear coupling)



Time series (asymmetric + nonlinear coupling)



Global sensitivity analysis



Main contributions:

- Simpler probabilistic model constructions
- Nontrivial insight into the behavior
- Important step for robustness and optimization problems

Sensitivity analysis based on variance decomposition

Mathematical Model:

$$\text{Quantity of Interest} = \mathcal{M}(\text{system parameters})$$

$$Y = \mathcal{M}(\mathbf{X}) \quad , \quad X_i \sim \mathcal{U}(0,1)$$

Hoeffding-Sobol decomposition:

$$Y = \mathcal{M}_0 + \sum_{i=1}^{k} \mathcal{M}_i\left(X_i\right) + \sum_{i<j}^{k} \mathcal{M}_{ij}\left(X_i, X_j\right) + \cdots + \mathcal{M}_{1\ldots k}\left(X_1 \ldots X_k\right)$$

An orthogonal decomposition in terms of conditional expectations:

- $\mathcal{M}_0 = \mathbb{E}\{Y\}$
- $\mathcal{M}_i(X_i) = \mathbb{E}\{Y \,|X_i\} - \mathcal{M}_0$
- $\mathcal{M}_{ij}(X_i, X_j) = \mathbb{E}\{Y \,|X_i, X_j\} - \mathcal{M}_i - \mathcal{M}_j - \mathcal{M}_0$
- $\ldots$

Sobol' indices

First-order Sobol' indices:

$$S_i = Var[\mathcal{M}_i(X_i)] \ / \ Var[\mathcal{M}(\mathbf{X})]$$

(quantify the additive effect of each input separately)

Second-order Sobol' indices:

$$S_{ij} = Var[\mathcal{M}_{ij}(X_i, X_j)] \ / \ Var[\mathcal{M}(\mathbf{X})]$$

(quantify interaction effect of inputs X_i and X_j)



I.M. Sobol *Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates.* **Mathematics and Computers in Simulation**, 55(1-3): 271-280, 2001.

PCE-based Sobol indices

Polynomial Chaos Expansion:

$$Y = \mathcal{M}(\mathbf{X}) \approx \sum_{\alpha \in \mathcal{A}} y_\alpha \, \psi_\alpha(\mathbf{X})$$

Analytical calculation of Sobol' indices:

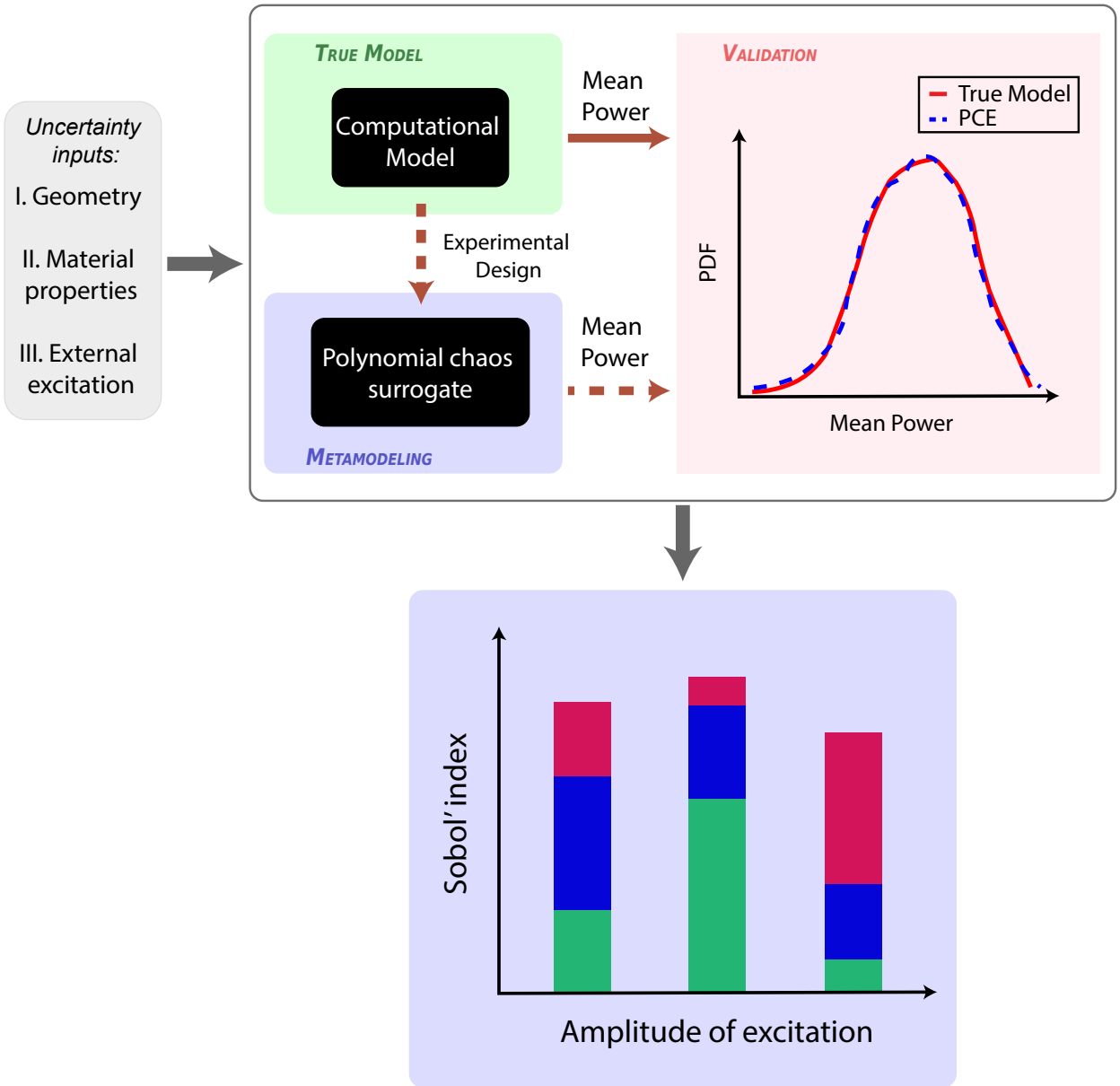
$$S_{\boldsymbol{u}} = \sum_{\boldsymbol{\alpha} \in \mathcal{A}_{\boldsymbol{u}}} y_{\boldsymbol{\alpha}}^2 \Bigg/ \sum_{\boldsymbol{\alpha} \in \mathcal{A} \setminus 0} y_{\boldsymbol{\alpha}}^2$$

$$\mathcal{A}_{\boldsymbol{u}} = \left\{ \boldsymbol{\alpha} \in \mathcal{A} : i \in \boldsymbol{u} \Longleftrightarrow \alpha_i \neq 0 \right\}$$



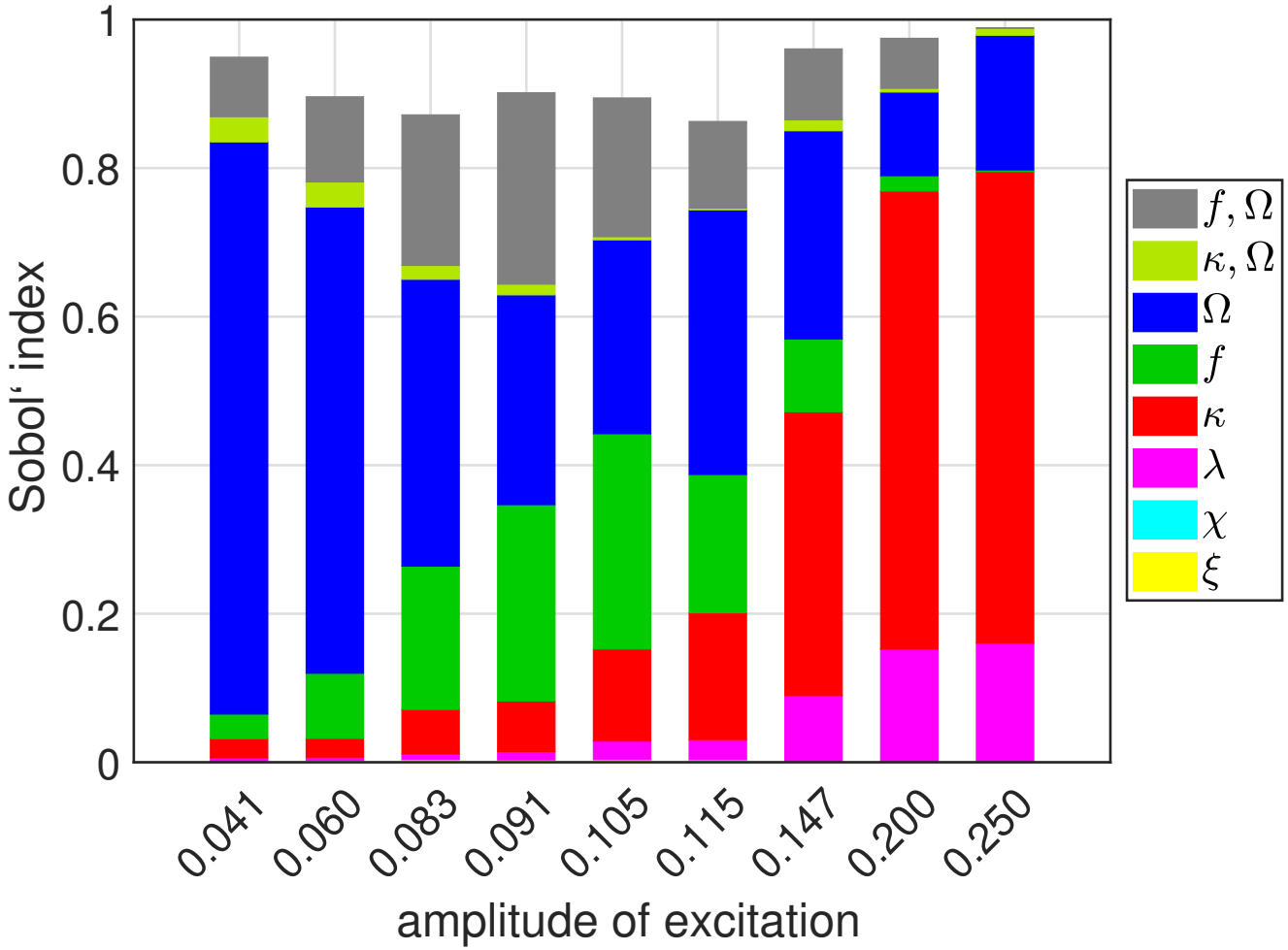
B. Sudret, *Global sensitivity analysis using polynomial chaos expansions.* **Reliability Engineering & System Safety, 2016**, 93(7): 964–979, 2008.

Global sensitivity analysis framework



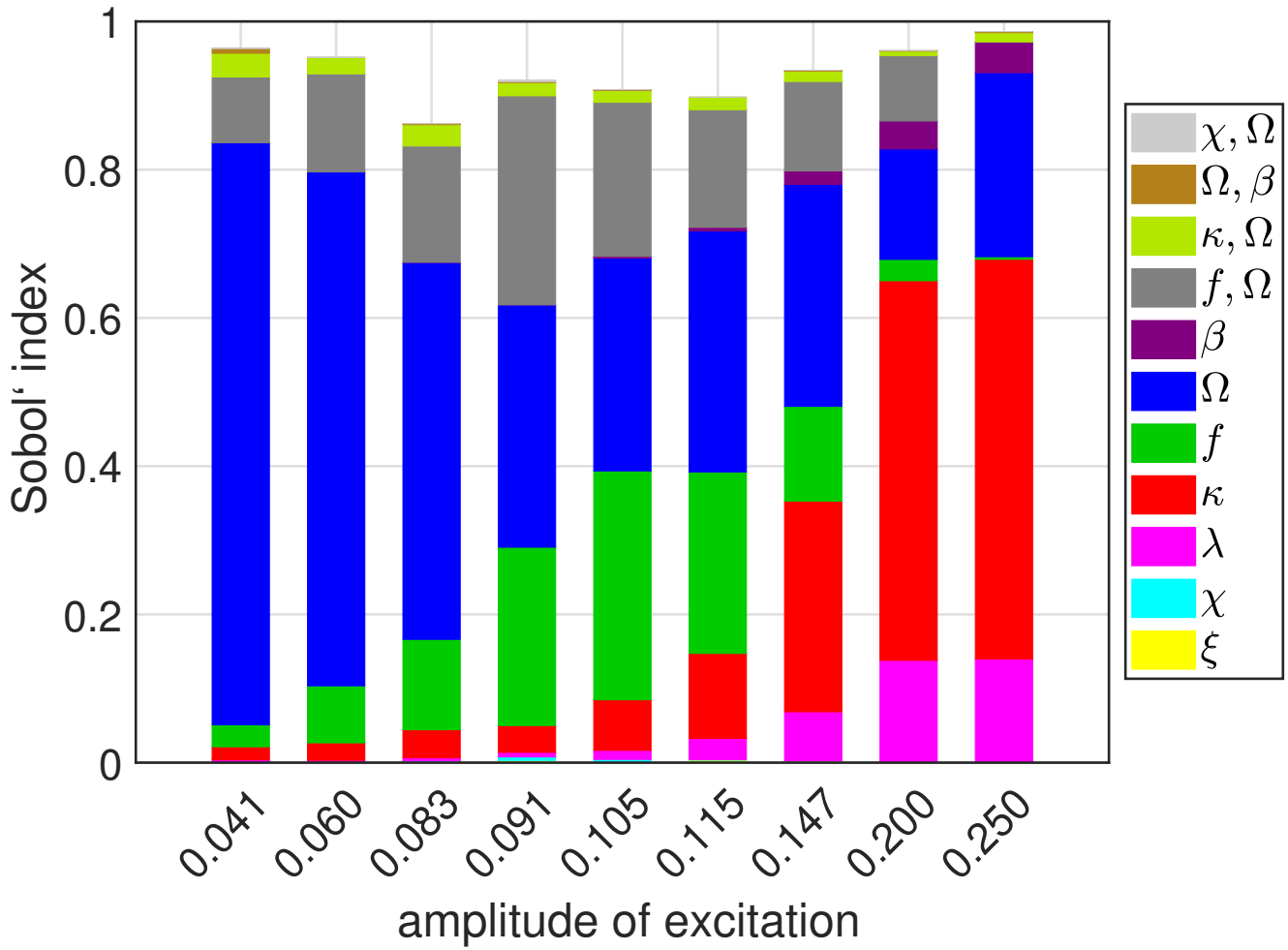
Mean power sensitivity (symmetric + linear piezo)

$$\beta = 0 \quad \phi = 0^{o} \quad f = 0.147 \quad \Omega = 0.8$$



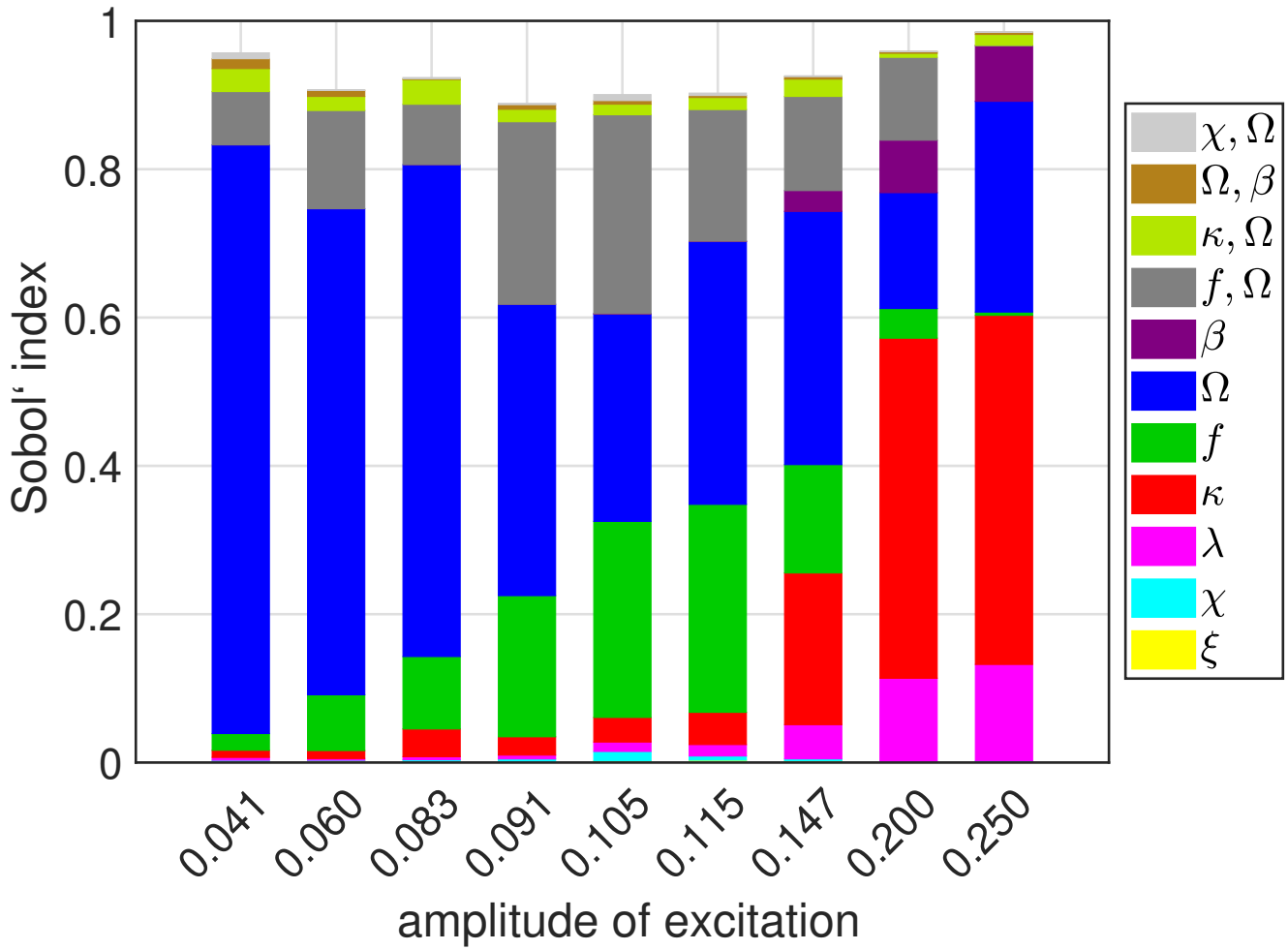
Mean power sensitivity (symmetric + nonlinear piezo)

$$\beta = 0.5 \quad \phi = 0^{o} \quad f = 0.147 \quad \Omega = 0.8$$



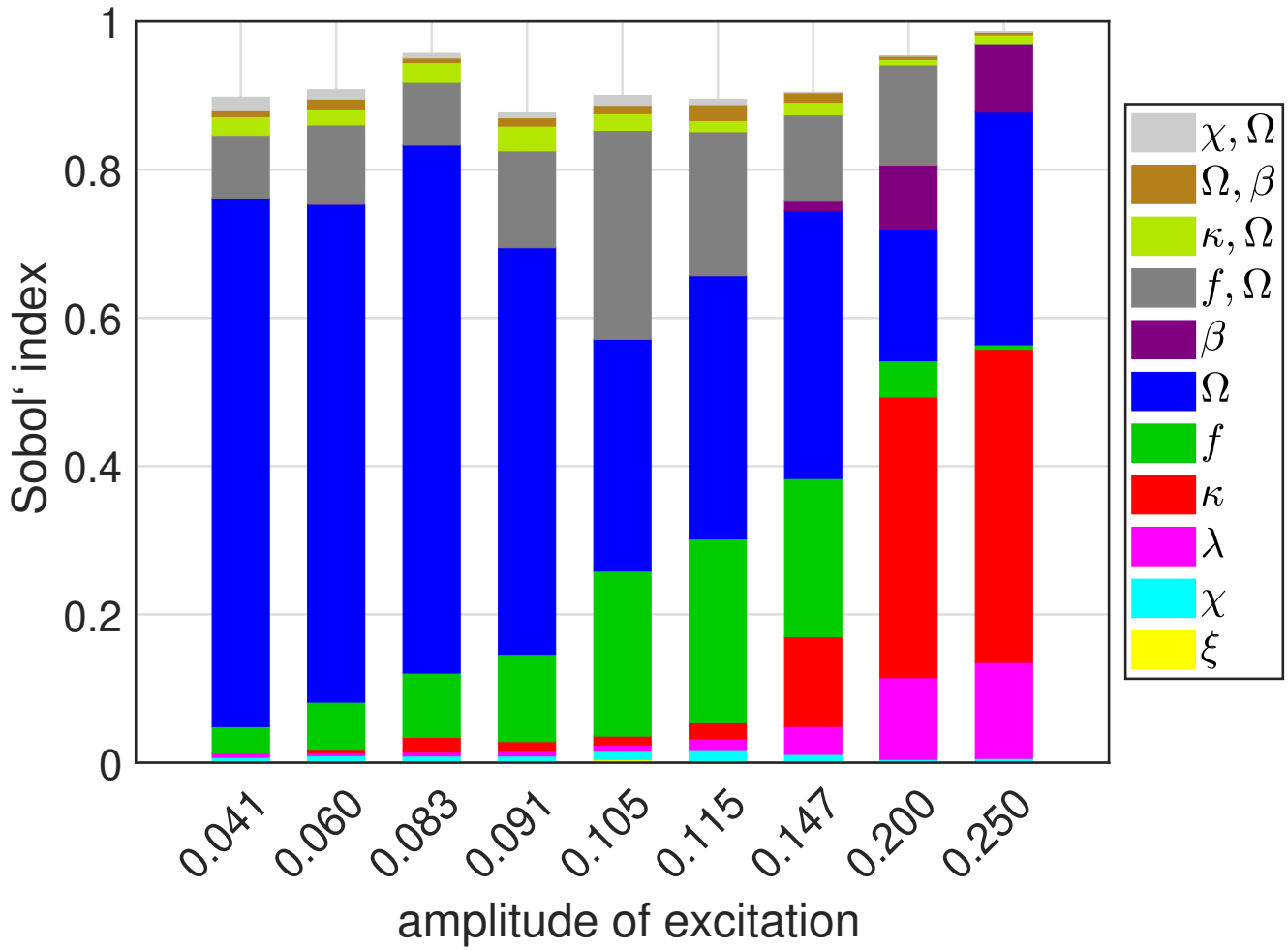
Mean power sensitivity (symmetric + nonlinear piezo)

$$\beta = 1.0 \quad \phi = 0^{o} \quad f = 0.147 \quad \Omega = 0.8$$



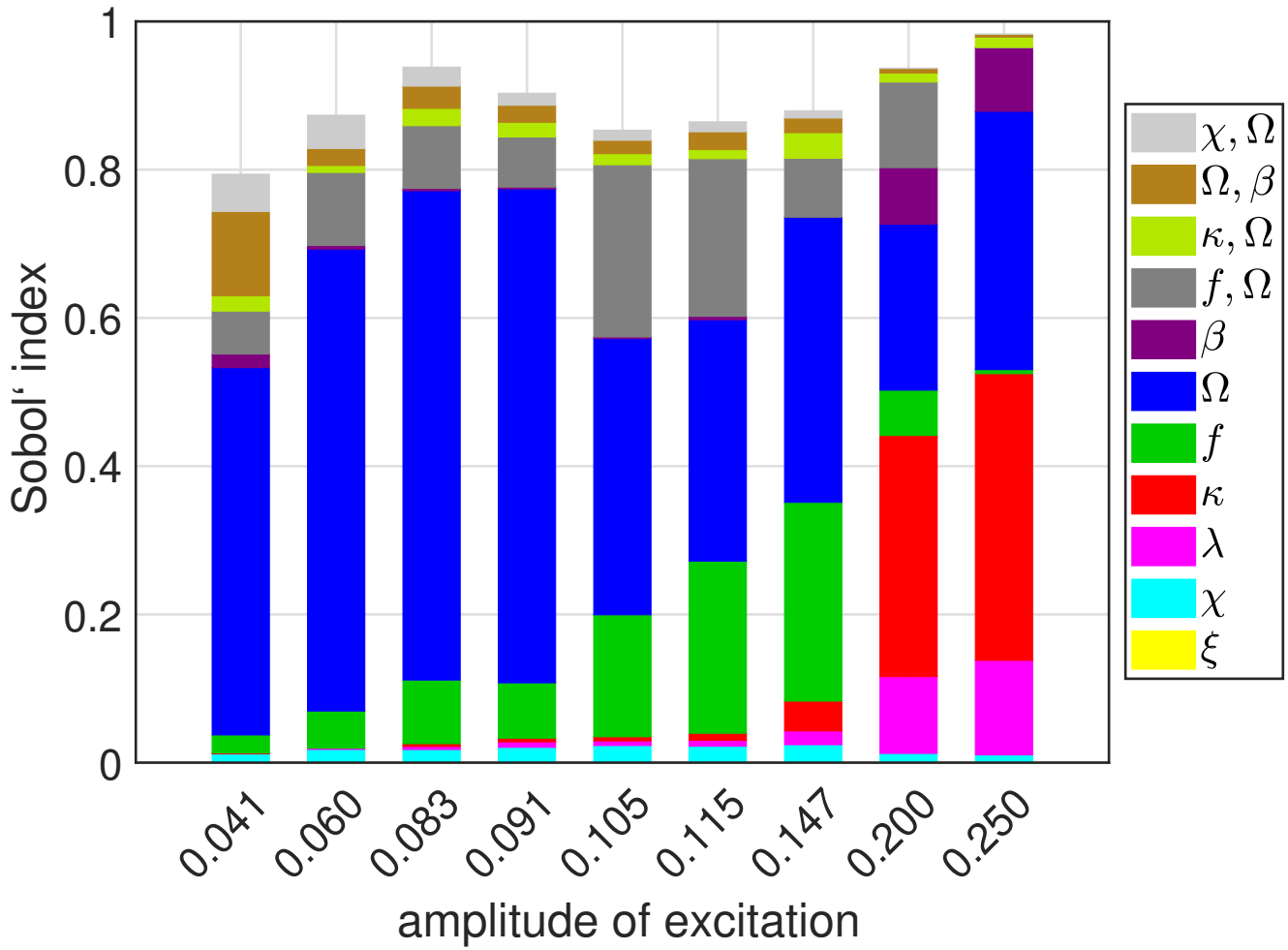
Mean power sensitivity (symmetric + nonlinear piezo)

$$\beta = 1.5 \quad \phi = 0^{o} \quad f = 0.147 \quad \Omega = 0.8$$



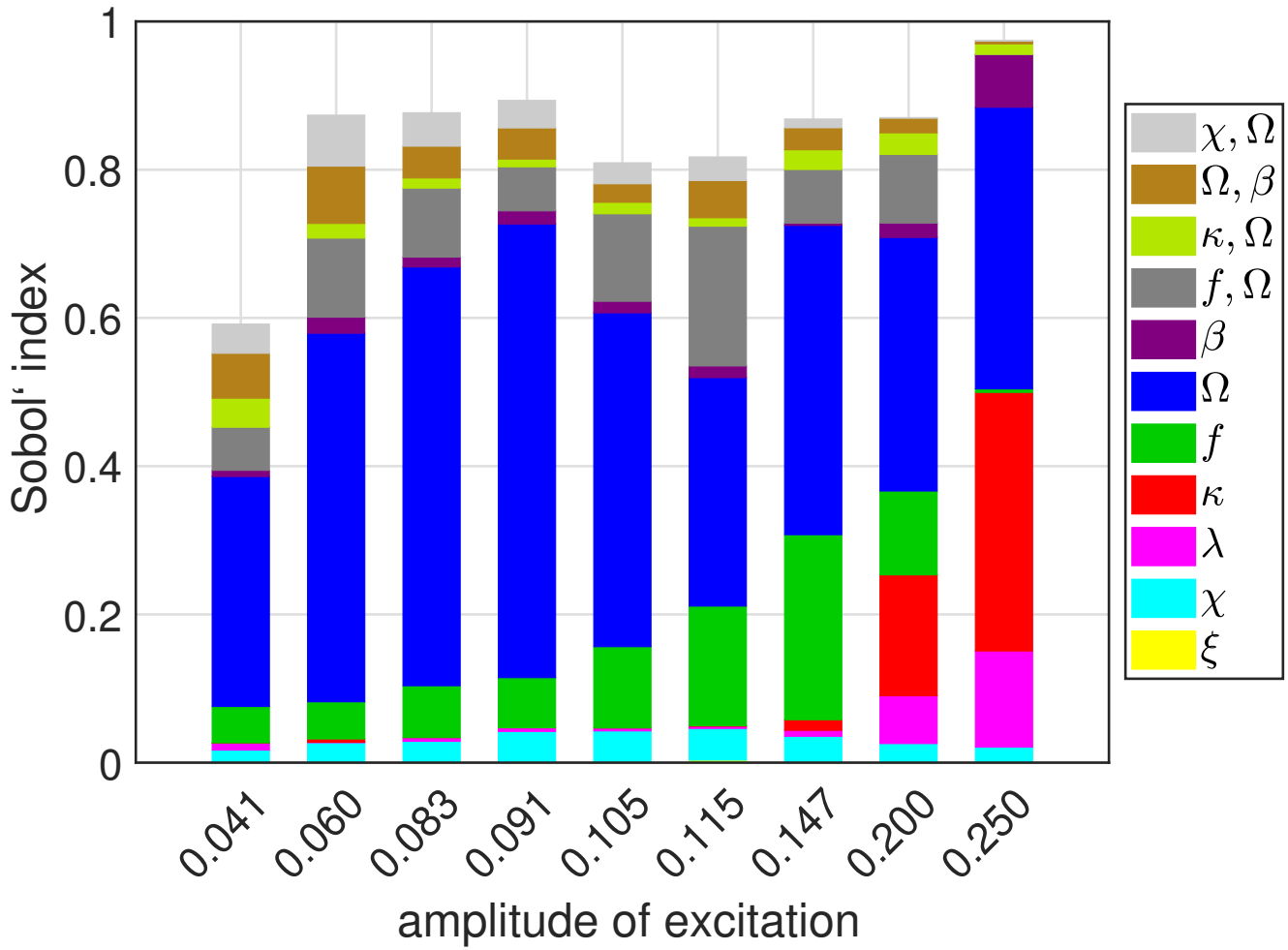
Mean power sensitivity (symmetric + nonlinear piezo)

$$\beta = 2.0 \quad \phi = 0^{o} \quad f = 0.147 \quad \Omega = 0.8$$



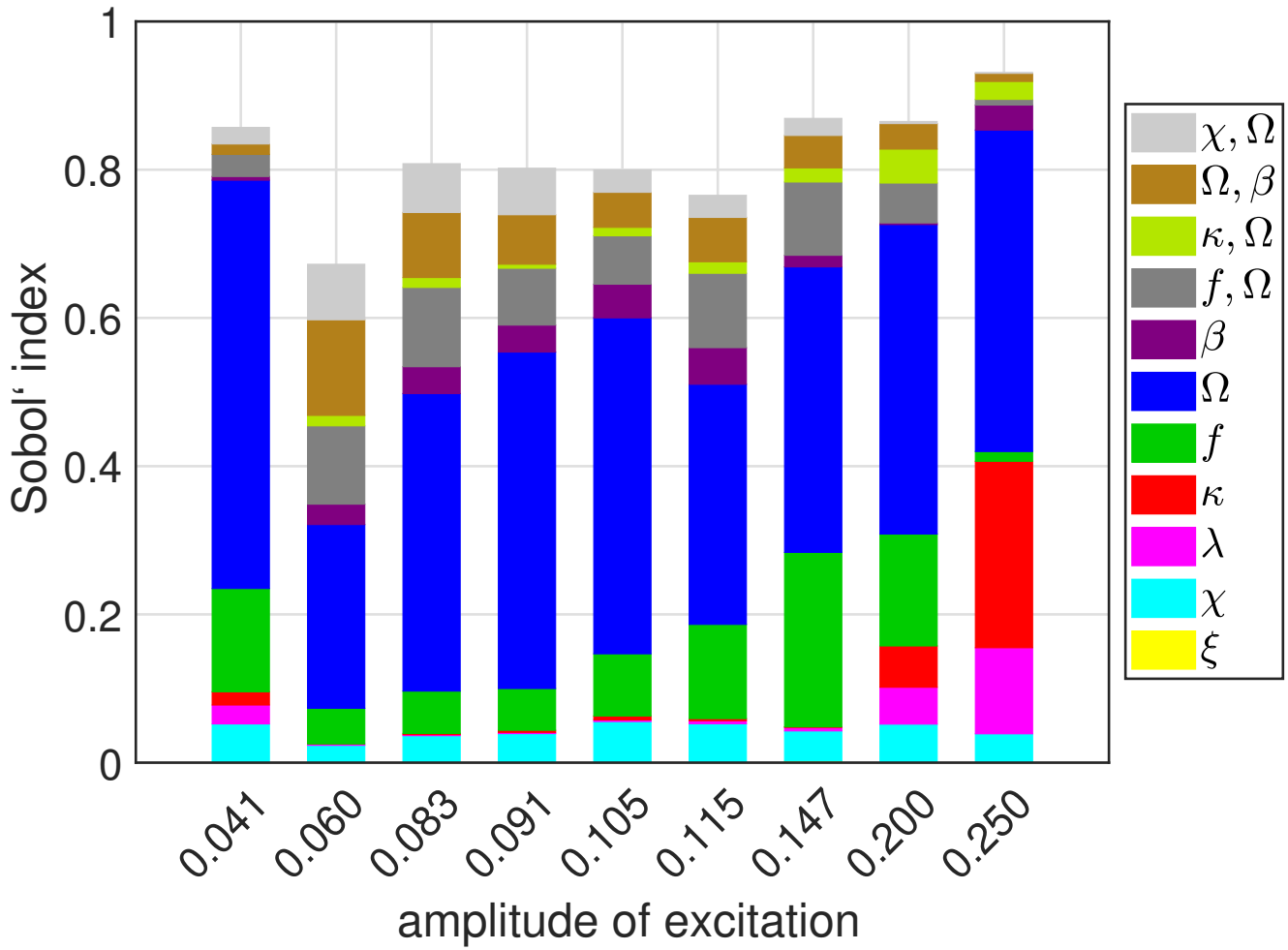
Mean power sensitivity (symmetric + nonlinear piezo)

$$\beta = 2.5 \quad \phi = 0^{o} \quad f = 0.147 \quad \Omega = 0.8$$



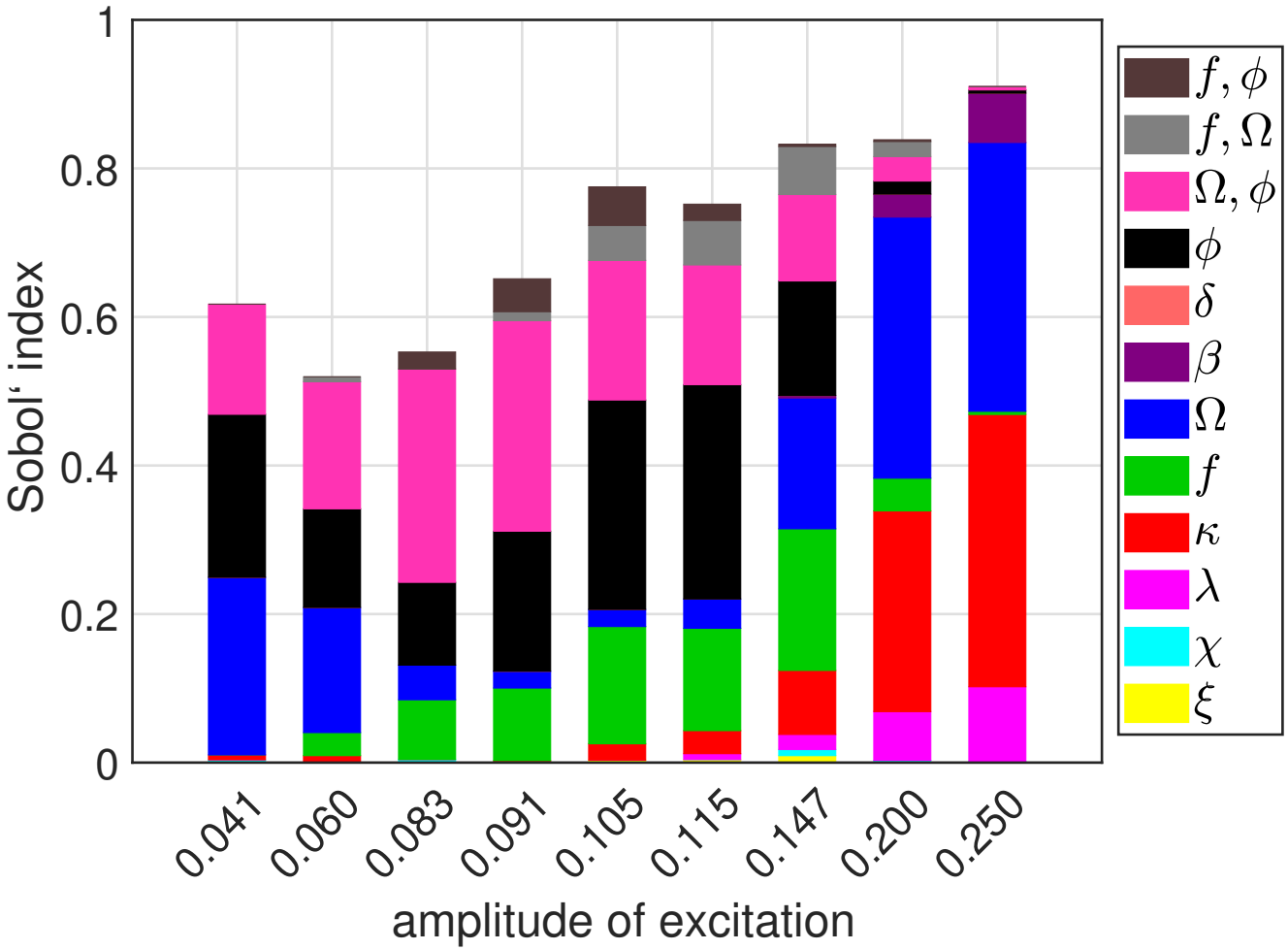
Mean power sensitivity (symmetric + nonlinear piezo)

$$\beta = 3.0 \quad \phi = 0^{o} \quad f = 0.147 \quad \Omega = 0.8$$



Mean power sensitivity (asymmetric + nonlinear piezo)

$$\beta = 1.0 \quad \phi \in [-15^{o}, 15^{o}] \quad f = 0.147 \quad \Omega = 0.8$$



Sensitivities in a nutshell

parameter	sensitivity	optimal condition
ξ	no	-
χ	no	-
λ	low	high energy
κ	high	high energy
f	high	low and middle energy
Ω	high	all spectrum
β	low	high values
δ	no	-
ϕ	high	low and middle energy

Conclusions and perspectives

Findings:

- The order of importance of the parameters changes a lot with vibration regime
- Frequency and amplitude of excitation, asymmetric bias angle, and piezoelectric coupling at the electrical domain are the most influential parameters that affect the mean power harvested
- Bias angle is a better parameter to optimize the harvested power for middle and low forcing amplitudes
- Piezoelectric electrical properties are better parameters to optimize the harvested power for high forcing amplitude

Ongoing research:

- Uncertainty quantification for parameters disturbances
- Experimental analysis

Experimental analysis (ongoing research)

Acknowledgments

Institutional support:





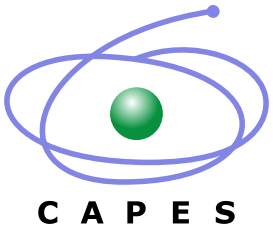


Financial support:









Thank you for your attention!

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J. P. Norenberg, A. Cunha Jr, S. da Silva, P. S. Varoto,
Global sensitivity analysis of (a)symmetric energy harvesters,
arXiv:2107.04647, 2021.
https://arxiv.org/abs/2107.04647

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